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Evaluation of Static Strength Margin of Thin-Wall Steel Cylinders on Coefficient of Permanent Expansion

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Abstract
Theoretical and experimental substantiation of application of method of permanent expansion for evaluation of static strength margin of steel cylinders have been presented. Common deformation curve and deformation plasticity theories are used. A possibility is provided for justified designation of a maximum allowable coefficient of permanent expansion used at periodic inspection of the cylinders.

Keywords: steel cylinders, hydraulic pressure tests, volume change, strength margin, periodic inspection

1. General provisions

The publication makes a hypothesis on common dependence between the coefficients of permanent expansion and coefficient of static strength margin for single-type cylinders. Theoretical and experimental arguments are provided in defense of it.

Hypothesis: Thin-wall steel cylinders, made of one steel grade and on one technology, independent on differences in volume, weight, wall thickness, outside diameter and length, will have the same dependence between current coefficient of static strength margin and coefficient of permanent expansion.
Notes: The cylinders shall have no defects detected by non-destructive testing methods. It means that if a cylinder is slowly loaded with rising internal pressure up to a limit state, i.e. its failure will take place due to exceeding a bearing capacity of metal. The dependence of current coefficient of static strength margin on coefficient of permanent expansion can be obtained experimentally as well as theoretically.

There is a certain spread on geometry parameters and mechanical properties in the process of manufacture of one-type cylinders. Some of the indices are regulated by the tolerances, other no. Usually only one maximum allowable limit is set. Melts of the same steel effect the considerable spread of the mechanical properties, apart of different emergency steels. As an example Figure 1 presents the results of the mechanical tensile tests of oxygen 40 l cylinders made on GOST 949-73 [1]. The maximum allowable values of yield point ($\bar{\sigma}_{YP}$) and ultimate strength ($\bar{\sigma}_U$) made 373 and 638 MPa, respectively. Conventional sign with a line over indicates that there are data on stress-strain diagram. Figure 1a shows the data during manufacture at Iron and Steel Illich Works for 2001-2008. The cylinder samples cut in axial direction from a witness shell were used. The sampling was made without repeated heat treatments. The tests were carried out on two samples of each batch, in total 447 batches. The batch contained not more than 400 cylinders [2]. The cylinders were manufactured mainly of Ds steel grade (TU 14-157-15). Figure 1b presents the actual deformation diagrams (see formula (4) below), obtained on the samples cut out in circular direction from 2 investigated cylinders made of Ds steel grade, and for a long time being in operation. The samples were cut out of non-deformed part under shoes after cylinders failure due to inner burst pressure. The diagrams show the information before deformation corresponding to ultimate strength, i.e. start of neck formation. A stress relationship is virtually constant in the area of insignificant plastic deformations.

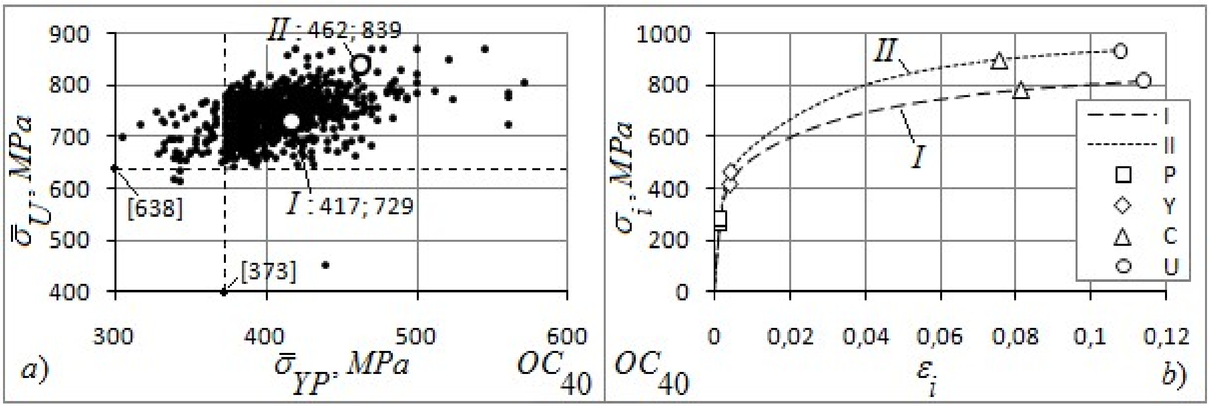

Figure 1. Mechanical properties of oxygen 40 liters cylinders:
a – in manufacture; b – actual diagrams of cylinders I and II deformation, plotted on tensile diagrams using dependencies (4),
P – proportionality limit σ_{PL} see GOST 1497-84 [3]; Y – conventional yield strength σ_{02};
C – value corresponding to cylinder limit state; U – ultimate strength σ_U. P, Y, U values of actual diagram of deformation are obtained from corresponding values on stress-strain diagram using dependencies (4).

Figure 1a shows sufficiently large spread of yield strength and ultimate strength in production of cylinder of one type at one plant. It is known that mechanical properties have normal distribution, therefore, using additional analysis it is possible to talk about ultimate strength. The yield strength is also far from this law. Virtually, even using Figure 1a it is possible to conclude that data on yield point in the area of its minimum allowable values on GOST were collected not very objectively. Besides, the tendency is traced that the higher the yield strength the higher ultimate strength is. As for strength, for example, a cylinder with larger diameter and smaller wall thickness, having higher mechanical properties, may be stronger, i.e. keep higher pressure. A process of cylinder operation can also make changes in strength, for example reducing wall thickness due to corrosion, etc. (following the European requirements, for example, corrosion is not allowed).

In manufacture and at periodic inspections of the cylinders the hydraulic pressure tests are carried out using internal test pressure (P_H). It depending on various requirements 1.25; 1.5 or 5/3 times exceeds working pressure (P_W). A series of reference documents (see, for example, [4, 5, 6, 7]) are sued for determination of volume change and coefficient of permanent expansion in process of such tests:

$$K_{PE} = \Delta W_{PE} / \Delta W_{TE} , \quad (1)$$

where ΔW_{TE} – total expansion of cylinder volume being under test pressure, ΔW_{PE} – permanent expansion – residual variation of cylinder volume after test pressure relieve to zero.

Changes of volume is mainly determined with help of «water jacket» method. A cylinder is placed in a close vessel with liquid having coming of it (vessel) graduated burette. At pressure loading the cylinder expandes and displaces liquid into a burette. The maximum allowable value K_{PE} is also set at 0.1 level for steel cylinders and 0.05 for composite cylinders. In Russia , for example, all cylinders are for 0.05.

In the USA, for instance, an elastic expansion is also determined in addition to the coefficient of permanent expansion following the recommendations of Compressed Gas Association:

$$\Delta W_{EE} = \Delta W_{TE} - \Delta W_{PE} . \quad (2)$$

Its maximum-allowable pressure is set based on limitation of averaged stresses in a cylinder wall, see [8]. An explanation of given formulae and experimental dependencies can be found in materials [9]. The elastic change of volume is related with geometry characteristics of the specific cylinder, permanent one is connected with value of plastic deformations, if they took place in cylinder wall at test pressure. The coefficient of permanent expansion is an integral characteristic and describes a level of these plastic deformations. It equals zero ($\Delta W_{PE} = 0$) in elastic area. The next information will show that it is the most sensitive to initial stages of plastic deformation, and then demonstrates weak reaction to further deformation. It can not be more than 1.

Real coefficient of cylinder static strength margin (n_B) – the burst ratio, is determined as the maximum pressure realized in cylinder (P_B) before its fracture referred to working one, and its maximum-allowable value is also regulated. For large spectrum of cylinders this coefficient on European and American requirements equals 2.4 and for post-Soviet area it is 2.6. If the coefficient is reduced by some reasons the period of further operation is limited. Real strength of the cylinder can only be determined by means of its fracture. Due to some different reasons n_B coefficient will have some spread even for cylinders belonging to one batch. It is clear that in the trial tests n_B is not lower than P_H / P_W relationship, but it is impossible to say what it is in fact for each specific cylinder. Using the method of volumetric expansion it can be assumed that the higher K_{PE} at that the closer the limit state is, i.e. P_B and , respectively, n_B will be lower.

The similar value of the maximum allowable coefficient is designated for different types of the cylinders independent on P_H / P_W relationship, and the minimum allowable coefficient of static strength margin. At that in the most cases the data of previous checks are not considered. More detailed information and references can be found in paper [10].

2. Theoretical backgrounds

Detailed workup of a theory stated below is considered in [11]. It also provides numerous experimental data, including extensive description and results of testing of full-scale sample used below. This publication states only main provisions.

For description of large deformations it is more reasonable to use a concept of logarithmic strain (ε), sometimes it is called actual and it can be expressed through current (l) and initial (l_o) linear dimensions or through common deformation ($\overline{e}=\Delta l/l_o$) in the following way:

$$\varepsilon=\int_{l_o}^{l} d\varepsilon=\int_{l_o}^{l}\frac{dl}{l}=\ln l-\ln l_o=\ln\frac{l}{l_o}=\ln\frac{l_o+\Delta l}{l_o}=\ln\left(1+\overline{e}\right). \qquad (3)$$

In this case $\Delta l=l-l_o$ is the finite increment.

Mathematic operation of the logarithmic strain is more reasonable due to elimination of additional members, which are inconvenient in transformations. Besides, they in contrast to common ones have an additively property. At small deformation the common and logarithmic strains are virtually indistinguishable.

A tension diagram is obtained at mechanical uniaxial tensile strength test: $\overline{e}=\Delta l/l_o$ is the common longitudinal deformation, $\overline{\sigma}=F/A_o$ is the stress, as a tensile force F, effecting the sample referred to the initial area of its working cross section A_o. This diagram will be re-plotted in actual strain diagram ε_i, σ_i using following dependencies:

$$\varepsilon_i=\varepsilon=\ln\left(1+\overline{e}\right),\ \sigma_i=\overline{\sigma}\left(1+\overline{e}\right). \qquad (4)$$

Then it can be used at multiaxial stressed state, but only till neck forming moment. Stress intensity (σ_i) in this case is already with considered dimension change. It should be noted that at such plotting of actual strain diagram we eliminate elastic deformations of volume taken by material, i.e. condition of incompressibility is accepted, Poisson's coefficient ν equals 0.5. A theorem of elastic unloading is kept in force. The actual strain diagram ε_i, σ_i is the material property and does not depend on type of stressed state. In general form, at complex stress-strain state Hook's law for elastic state has the following form: $\sigma_i=E\varepsilon_i$. It should be noted that further the stresses are determined not by initial, but on actual, current dimensions, see, for example, formula (6). Hereinafter only such stresses and logarithmic strains will be used.

The calculation is carried out on thin-wall theory, i.e. wall thickness is small in comparison with diameter. Denoting the initial values of radius of a middle surface, wall thickness and cylinder length through r_o, s_o, l_o, the current ones varying in the process of deformation under loading can be expressed in accordance with formula (3) in the following way:

$$r=r_o e^{\varepsilon_t},\ s=s_o e^{\varepsilon_s},\ l=l_o e^{\varepsilon_z}, \qquad (5)$$

where $\varepsilon_t, \varepsilon_s, \varepsilon_z$ are the deformations of cylinder in circumferential, on wall thickness and axial directions (main deformations).

Changes of dimensions in the elastic area are insignificant, therefore, they are usually neglected, whereas in the field of plastic deformations the changes are considerable and they should not be neglected.

Inner pressure acting on the bottoms of cylinders creates an axial force and, respectively, axial stresses. The radial stresses in the cylinder are considered being equal 0, i.e. plane stressed state is taken. Cylindrical part located in plastic field, as it will be shown below, based on incompressibility assumption, will be under conditions of plane deformation state, i.e. axial deformations equal zero. Studying deformation of the thin-wall steel cylinder out of elasticity limits several plasticity theories can be used. For the cylinder under effect of inner pressure, independent on selected plasticity theory, the main stresses in the cylinder wall, namely circumferential, axial and radial ones, are expressed in the following way:

$$\sigma_t = Pr/s,\ \sigma_z = \sigma_t/2,\ \sigma_r = 0\,. \tag{6}$$

The following is received when determining the stress intensity as an equivalent stress on maximum-distortion-energy theory, namely hypothesis of specific potential energy of shape change, and substituting the stresses from (6):

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_t - \sigma_z\right)^2 + \left(\sigma_z\right)^2 + \left(\sigma_t\right)^2} = \sqrt{\sigma_t^2 - \sigma_t\sigma_z + \sigma_z^2} = \frac{\sqrt{3}}{2}\sigma_t = \frac{\sqrt{3}Pr}{2s}\,. \tag{7}$$

In plastic region as well as in elastic the stress intensity is proportional to its constituents and these constituents between themselves are also proportional and in turn proportional to pressure.

Also it should be noted that due to geometry and power symmetry, the tangential stresses on main areas, which are parallel and normal to the axis, equal zero. The main axes of stressed state keep their direction in loading of cylinder with inner pressure. The stresses are proportional between themselves and rise proportionally to pressure. Such a type of deformation is called common deformation, and loading is the common loading. In the case of common loading, the intensity of increment of plastic deformations equals increment of intensity of plastic deformations. In such a case when describing large plastic deformations it is reasonable to use deformation theory of plasticity (theory of small elasto-plastic deformations). The process of loading at small as well as at large deformations is simple. The linear deformations corresponding to main stresses are also main ones. For the areas significantly distant from the bottom, the radial displacements do not depend on location along the axis. Therefore, it is possible to study a part of cylinder, cut out by two sections normal to cylinder axis, and compile the equations of shell element equilibrium selected from this part by toe radial sections making between themselves some angle. It is true for elastic as well as plastic stage of loading. The deformations being determined on different theories of ductility can vary. Description of plastic deformations of the cylinder, for example using plastic flow theory (flow theory), provides the same results as based on deformation plasticity theory. The flow theory is used in the cases when the stresses are not proportional between themselves at loading, i.e. loading is not simple [12]. The intensity of increment of plastic deformations in general case does not equal the intensity increment of plastic deformations. Equality is a special case and it shall be proved.
In construction of deformation theory of plasticity as well as theory of plastic flow the equations still contain an elasticity module and Poisson's ratio in the elastic region. They allow getting the elastic solutions using the dependencies obtained for plastic state (general solutions) at

insignificant deformations. At plastic deformations the elastic constituent in the equations is small. The actual Poisson's ratio in the field of plastic deformation tends to 0.5 [13]. Under loading conditions there are only elasticity equations.

Breaking down the main deformations through the stresses [12] and expressing the stresses through their intensity, the main deformations for the cylinder are obtained:

$$\varepsilon_t = \frac{\sigma_i}{2\sqrt{3}}\left(\frac{1-2\nu}{E}+\frac{3}{E_S}\right),\ \varepsilon_s = \frac{\sqrt{3}\sigma_i}{2}\left(\frac{1-2\nu}{E}-\frac{1}{E_S}\right),\ \varepsilon_z = \frac{2\sigma_i}{\sqrt{3}E}\left(\frac{1}{2}-\nu\right), \tag{8}$$

where E_S being the elasticity secant modulus on actual deformation diagram is the variable value.

It can be seen from this that the axial deformations are always elastic and the deformations on wall thickness are negative. It should be noted if the expression contains the elasticity modulus (E) or the same $\nu \neq 0.5$, than the secant modulus should not be described as σ_i / ε_i, since in contrast to stress intensity, the intensities of deformations are different for the cylinder and on actual deformation diagram. Assuming $\nu = 0.5$ for the cylinder, they completely match, only in this case it is possible to accept $E_S = \sigma_i / \varepsilon_i$ and perform further transformations.

Describing the deformation intensity by definition using the dependencies (8) for cylinder we get:

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{\left(\varepsilon_t - \varepsilon_z\right)^2 + \left(\varepsilon_z - \varepsilon_s\right)^2 + \left(\varepsilon_s - \varepsilon_t\right)^2} = \frac{\sigma_i}{3}\left(\frac{2\nu-1}{E}+\frac{3}{E_S}\right). \tag{9}$$

Substituting in formulas (8, 9) E_S for E, the dependence of calculation of elastic deformation is obtained. The dependencies are significantly simplified, if suppose that the material is incompressible, i.e. assume $\nu = 0.5$. It follows form this that $\varepsilon_i = \frac{\sigma_i}{E_S} = \frac{2}{\sqrt{3}}\varepsilon_t$, $\varepsilon_s = -\varepsilon_t$ and $\varepsilon_z = 0$. For this case it is possible to show that $r \cdot s = r_o \cdot s_o$ and $\sigma_i \cdot \varepsilon_i = \sigma_t \cdot \varepsilon_t$. In general, if it is taken that $\nu = 0.5$ the solutions in the plastic region are more approached to reality, but at small deformations these solutions do not automatically transfer into elastic solutions, rather they transfer to such elastic solutions, where Poisson's ratio is also equals 0.5.

In the plastic region in contrast to elastic one, the deformation intensity is not proportional to stress intensity. Also the deformation intensity is not proportional to each of the linear deformations. At $\nu = 0.5$ the deformation intensity becomes proportional to linear deformations except for axial ones, which in this case equal zero. And it is still not proportional to stress intensity.

Since logarithmic deformations in contrast to common ones have the additivity property, then after release of internal pressure resulting in plastic deformations, the residual deformations can be expressed as a difference between complete (ε_i) and elastic (ε_{ie}) ones. Naturally that release is elastic:

$$\varepsilon_{ires}=\varepsilon_i-\varepsilon_{ie}=\left(\frac{\sigma_i}{3}\left(\frac{2\nu-1}{E}+\frac{3}{E_S}\right)\right)-\left(\frac{2\sigma_i(1+\nu)}{3E}\right)=\sigma_i\left(\frac{1}{E_S}-\frac{1}{E}\right). \quad (10)$$

It is clear that in this case σ_i value corresponds to stress intensity value during loading. It can be seen that this expression does not depend on Poisson's ratio and completely match with Odqvist's parameter, i.e. residual deformations on actual deformation diagram. In other words in this case the residual deformations in the cylinder already match with the residual deformations on the actual deformation diagram. The main deformations can be treated in the similar way. It follows from this that $\varepsilon_{tres}=\frac{\sqrt{3}}{2}\varepsilon_{ires}$, $\varepsilon_{sres}=-\varepsilon_{tres}$, $\varepsilon_{zres}=0$ as well as that $r_{res}\cdot s_{res}=r_o\cdot s_o$. These data indicate that the residual deformations do not depend on Poisson's ratio.

For the case of simple loading the deformation intensity can be described as sum of elastic and plastic constituents $\varepsilon_i=\varepsilon_{ie}+\varepsilon_{ip}$, where $\varepsilon_{ip}=\varepsilon_{ires}$. The same will be true for increments $d\varepsilon_i=d\varepsilon_{ie}+d\varepsilon_{ip}$. Under ε_{ie} can be treated as a value of elastic relief in each point. The larger the deformation, the more the stress is and the higher the value of elastic relief will be. It is interesting to note that the plastic constituent of deformations as well as its increment does not depend on Poisson's ratio.

Using the current and initial dimensions, let's describe a change of cylinder volume as change of volume acquired by cylinder shell:
$\Delta W=\pi r^2 l-\pi r_o^2 l_o=\pi r_o^2 l_o\left(e^{2\varepsilon_t+\varepsilon_z}-1\right)=W_o\left(e^{2\varepsilon_t+\varepsilon_z}-1\right)$. Describing the main deformations the general solution is obtained:

$$\Delta W=W_o\left(\exp\left(\frac{\sigma_i}{\sqrt{3}}\left(\frac{2-4\nu}{E}+\frac{3}{E_S}\right)\right)-1\right)=W_o\left(\exp\left(\sqrt{3}\varepsilon_i\frac{\frac{2-4\nu}{E}+\frac{3}{E_S}}{\frac{2\nu-1}{E}+\frac{3}{E_S}}\right)-1\right). \quad (11)$$

In this case $W_o=\pi r_o^2 l_o$ is the initial volume, however, it can be determined directly, for example, as internal, and ε_i is determined by formula (9), and not on actual deformation diagram. The corresponding is obtained in the elastic region:
$\Delta W_e=\Delta W=W_o\left(\exp\left(\frac{\sigma_i(5-4\nu)}{\sqrt{3}E}\right)-1\right)=W_o\left(\exp\left(\frac{\varepsilon_i\sqrt{3}(5-4\nu)}{2(1+\nu)}\right)-1\right)$, and at $\nu=0.5$ it follows from the general solution $\Delta W=W_o\left(\exp\left(\frac{\sqrt{3}\sigma_i}{E_S}\right)-1\right)=W_o\left(\exp\left(\sqrt{3}\varepsilon_i\right)-1\right)$.

It can be seen from general solution (11) that it is impossible to expand the complete change of volume under load for elastic (ΔW_e), releasing after relief, and plastic (ΔW_p) components. Conventionally supposing that $\Delta W=\Delta W_e+\Delta W_p$, plastic change of volume can be presented as $\Delta W_p = \Delta W - \Delta W_e$. Naturally, that rise of plastic deformations ΔW_e, the same as ΔW provokes increase of ΔW_p, but in less degree than the last. The plastic constituent of volume change is related with the residual deformations. $\Delta W = f\left(\varepsilon_i\right)$ dependence is close to linear one. It can be shown that expression for ΔW_p does not virtually depend on Poisson's ratio. But it should be noted that the difference values of ΔW and ΔW_e at $\nu \neq 0.5$ and $\nu = 0.5$ can have significant variation between themselves. As for the residual deformations and their intensity, they demonstrate complete matching and constituents of the difference at $\nu \neq 0.5$ and $\nu = 0.5$ can also be considerably different between themselves.

Determination of residual change of cylinder volume can be done easier. Let's write it as change of volume of cylinder shell using residual and initial dimensions, see above. Since we know the residual circumferential deformations, which can also be expressed through residual deformation intensity, at that we know that the axial residual stresses equal zero, the next is obtained:

$$\Delta W_{res} = W_o\left(\exp\left(\sqrt{3}\varepsilon_{ires}\right) - 1\right) = W_o\left(\exp\left(\sqrt{3}\sigma_i\left(\frac{1}{E_S} - \frac{1}{E}\right)\right) - 1\right). \tag{12}$$

ΔW_{res} value is very close to ΔW_p and found as difference. It can be assumed that $\Delta W = \Delta W_{res} + \Delta W_e$. Altogether for determination of volume change several dependencies can be proposed, naturally the results will be somewhat different, but the general principle is not violated at that.

Having the information about volume change, for example at testing in water jacket (WJ), it is possible to determine circular deformations and deformation intensity. Adopting $\nu = 0.5$, thus eliminating secant modulus, the next is obtained:

$$\varepsilon_i = \left\{\frac{2}{\sqrt{3}}\varepsilon_t\right\} = \frac{1}{\sqrt{3}}\ln\left(\frac{\Delta W}{W_o} + 1\right). \tag{13}$$

Substituting complete and residual volumetric expansions obtained in tests with WJ in formula (13) instead of ΔW, we get a complete and residual deformation intensity.

Knowing the circular deformations and deformations on wall thickness, it is possible to determine the pressure corresponding to current loaded state. Since (using formula (5)):

$\frac{r}{s} = \frac{r_o e^{\varepsilon_t}}{s_o e^{\varepsilon_s}} = \frac{r_o}{s_o}\exp\left(\frac{\sigma_i}{\sqrt{3}}\left(\frac{3}{E_S} - \frac{1-2\nu}{E}\right)\right) = \frac{r_o}{s_o}\exp\left(\sqrt{3}\varepsilon_i\right)$, then

$$P = \frac{2\sigma_i s}{\sqrt{3}r} = \frac{2\sigma_i s_o}{\sqrt{3}r_o\exp\left(\frac{\sigma_i}{\sqrt{3}}\left(\frac{3}{E_S} - \frac{1-2\nu}{E}\right)\right)} = \frac{2\sigma_i s_o}{\sqrt{3}r_o\exp\left(\sqrt{3}\varepsilon_i\right)}. \tag{14}$$

Let's remind that deformation intensity in this case is taken for cylinder, it somewhat different from deformation intensity on the actual deformation diagram. At $\nu = 0.5$, the deformation intensity is taken already on actual deformation diagram. So, in this case the pressure is somewhat lower. The formula for pressure can be reduced to stresses and residual deformations. It can be seen from (14) that connection of pressure with actual deformation diagram is performed through geometry parameters of the cylinder r_o, s_o. The rest considered here values can be connected indirectly without applying geometry parameters. $\Delta W, P$ dependence can be made using ε_i, σ_i diagram and geometry parameters of the cylinder.

Let's consider the condition of buckling at plastic deformation. Expression $P = 2s\sigma_i / \sqrt{3}r$ is differentiated, see formula (14). Since the current values of radius and wall thickness are not constant, we have $dP = \frac{2}{\sqrt{3}}\left(\frac{sd\sigma_i}{r} + \frac{\sigma_i ds}{r} - \frac{\sigma_i sdr}{r^2}\right)$. It is clear that deformation provokes decrease of wall thickness ($ds < 0$), rise of radius of middle surface ($dr > 0$), and regardless the fact that increment of stress intensity is more than zero, but increase of plastic deformations decrease these increments. Therefore, dP values becomes lower and lower. As it is shown in [13], when the pressure stops to increase at processing medium forcing, i.e. dP becomes equal 0, then the shell with bottoms lose the process of stable plastic deformation, forming local wall thinning, and it failures due to pressure loss. However, theoretically, if this does not take place, then further uniform deformation will provoke pressure drop. It follows from expression $dP = 0$ that expression in brackets shall equal zero. Set to zero, preliminary multiplying by r and rearranging, we find: $d\sigma_i = \sigma_i\left(\frac{dr}{r} - \frac{ds}{s}\right)$. In the brackets there are the increments of circular deformations and deformations on wall thickness, i.e. $d\sigma_i = \sigma_i\left(d\varepsilon_t - d\varepsilon_s\right)$. At $\nu = 0.5$, we get $d\varepsilon_s = -d\varepsilon_t = -\sqrt{3}d\varepsilon_i / 2$. It follows form this that $d\sigma_i = \sigma_i\left(d\varepsilon_t - d\varepsilon_s\right) = \sigma_i 2d\varepsilon_t = \sqrt{3}\sigma_i d\varepsilon_i$ As a result the correspondence to condition of uniform plastic deformation loss is obtained on the actual deformation diagram [13]:

$$\sigma_i = \frac{1}{\sqrt{3}} \cdot \frac{d\sigma_i}{d\varepsilon_i}. \qquad (15)$$

Describing complete increments of the main deformations and their intensity, it is possible to proceed to the expression $d\varepsilon_t - d\varepsilon_s = \sqrt{3}d\varepsilon_i$. As a result the same expression is obtained for loss of stable plastic deformation as in $\nu = 0.5$. But in this case, increment of deformation intensity is based on the diagram for cylinder at $\nu \neq 0.5$.

Let's imagine some point on the diagram ε_i, σ_i. It is possible to found for it the corresponding coefficients of permanent expansion and static strength margin. Describing the coefficients of permanent expansion through calculation variations of volumes, we have: $K_{PE} = \frac{\Delta W_p}{\Delta W} = \frac{\Delta W - \Delta W_e}{\Delta W} = 1 - \frac{\Delta W_e}{\Delta W}$. Substituting here found above changes of volumes, the first formula (16) is obtained. Instead of ΔW_p it was possible to use ΔW_{res} on formula (12) and ΔW can be described as $\Delta W_{res} + \Delta W_e$. The result does not virtually change at that. The current coefficient of strength margin is the relationship of the maximum pressure to pressure in i-th point $n_{Bi}=P_B/P_i$. Implying the index m for values of corresponding maximum pressure, which was found using the condition of loss of uniform plastic deformation, after reduction, the second formula (16) is obtained.

$$K_{PE}=1-\frac{\exp\left(\frac{\sigma_i(5-4\nu)}{\sqrt{3}E}\right)-1}{\exp\left(\frac{\sigma_i}{\sqrt{3}}\left(\frac{2-4\nu}{E}+\frac{3}{E_S}\right)\right)-1},\ n_{Bi}=\frac{P_B}{P_i}=\frac{\sigma_{im}\exp\left(\frac{\sigma_i}{\sqrt{3}}\left(\frac{3}{E_S}-\frac{1-2\nu}{E}\right)\right)}{\sigma_i\exp\left(\frac{\sigma_{im}}{\sqrt{3}}\left(\frac{3}{E_{Sm}}-\frac{1-2\nu}{E}\right)\right)}. \qquad (16)$$

In elastic area $K_{PE}=0$ and at $\nu=0.5$: $K_{PE}=1-\frac{\exp\left(\frac{\sqrt{3}\sigma_i}{E}\right)-1}{\exp\left(\frac{\sqrt{3}\sigma_i}{E_S}\right)-1}$; $n_{Bi}=\frac{\sigma_{im}\exp\left(\frac{\sqrt{3}\sigma_i}{E_S}\right)}{\sigma_i\exp\left(\frac{\sqrt{3}\sigma_{im}}{E_{Sm}}\right)}$.

The calculation current value of coefficient of permanent expansion at $\nu=0.5$ is little bit larger than at $\nu<0.5$. As for the calculation current value of coefficient of strength margin the situation is vice verse. In general, when determining the current coefficient of strength margin, the Poisson's coefficient has virtually no effect and it is possible to assumed that $\nu=0.5$. Error at that in comparison with calculation at $\nu=0.3$ for example is negative and on absolute value makes less than 0.1%.

From expressions (16) it can be seen that theoretical K_{PE} and n_{Bi} do not depend on geometry parameters and volume of cylinder in thin-wall its presentation. They are determined by properties of actual deformation diagram and essence of cylinder shape with bottoms creating no edge effect. Nevertheless, the graphical dependence $n_{Bi}=f(K_{PE})$ has the same property. There are also all backgrounds for plotting the theoretical curves of other types of cylinders, for example, ball-shaped and composite.

3. Experiment

Let's determine experimental dependence $n_{Bi}=f(K_{PE})$ for one cylinder. For this, the process of cylinder loading with internal pressure up to its fracture is divided on stages. The maximum pressure P_i of each i-th stage exceeds the maximum pressure of previous (i-1)-th. Then, it resets to zero. Since, loading is carried out in stages, then complete and residual change of volume, necessary for determination of complete coefficient of permanent expansion, shall be taken considering the previous stages. In such a manner as if the loading took place at one time. Figure 2 provides geometry interpreting of such loads.

If measure the coefficient of residual permanent expansion on definition: $(K_{PE})_i=(\Delta W_{PE})_i/(\Delta W_{TE})_i$, at each i-th stage of cylinder loading with inner pressure where change of volume is only related with this stage, then complete K_{PE} at loading at one time will not equal the sum of stage-by-stage coefficients, it will be less:

$$K_{PE}=\frac{\Delta W_{PE}}{\Delta W_{TE}}=\frac{\left(\Delta W_{PE}\right)_i+\sum\limits_{n=1}^{i-1}\Delta W_{PE}}{\left(\Delta W_{TE}\right)_i+\sum\limits_{n=1}^{i-1}\Delta W_{PE}}<\sum_{n=1}^{i}\left(K_{PE}\right)_i . \qquad (17)$$

In general, if $K_{PE}<1$ then $\sum\left(K_{PE}\right)_i$ can be much more than one. The formula for current coefficient of strength margin will remain the same, i.e. $n_{Bi}=P_B/P_i$. P_B in this case is already taken from the test results.

Experimental data presented below were obtained in testing the sample (cylinder) with conventional sign I1. The sample was made of 219×6 pipe from steel 20 with welded flat bottoms. Working and hydraulic test pressure was taken equal 10 and 15 MPa, respectively. The sample at initial stages of testing took place in WJ manufactured at E. O. Paton Electric Welding Institute, then, it was fractured already out of it. The video of testing process is given in [14]. Figure 2 provides the experimental dependencies of complete and residual volumetric expansions from pressure obtained in sample I1 testing in the water jacket.

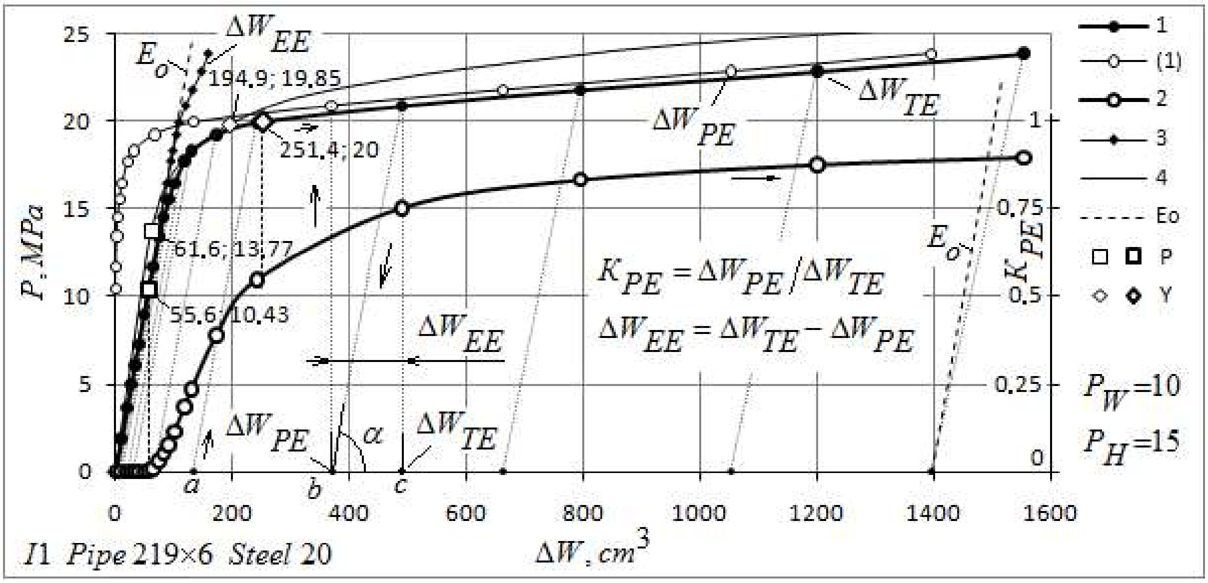

Figure 2. Experimental dependencies of complete and permanent volumetric expansions from pressure received in testing the sample I1 in water jacket:

1 (1) – complete and residual change of volume determined on WJ; 2 – recalculated on formula (17) K_{PE} – on right scale; 3 – difference between complete and residual volume change; 4 – calculation dependence at $\nu=0.5$; E_0 is the linear dependence plotted for elastic area before start of plastic deformations. Markers in bold P and Y designate pressure $\{P_Y\}$ and P_Y, and not bold are the calculation values corresponding σ_{PL} and σ_{02}.

The pressures used below were designated in the following way, namely: $\{P_Y\}$ is the pressure of yield beginning – cylinder volume stop returning into initial state; P_Y is the yield pressure determined as an inflection point of diagram of inner pressure loading; $P_{wat\ j}$ is the pressure, till which the sample was tested in WJ; P_B is the maximum pressure carried by sample; $\{P_B\}$ is the fracture pressure.

Values $(\Delta W_{TE})_i$ and $(\Delta W_{PE})_i$ for i-th stage of loading equal the distances (c-a) and (b-a), see Figure 2. It is clear that ΔW_{EE} in these systems of coordinates is invariant. $ctg(\alpha)$ value, see Figure 2, is the elastic volumetric compliance (β). And this value rises with increase of plastic deformations, [9]. There is correspondence of $\beta_o = ctg(\alpha_o) = 5{,}329$ for straight line E_o, which on Figure 2 is given on the left, and, for comparison, on the right. β is constant in the field of elastic deformations. It insignificantly increases to 5.45 from proportional limit stress to yield point. Then, it demonstrates significant rise and to the end of testing in WJ it makes 6.67, moreover the rise is virtually linear. The elastic change of volume can also be presented in the following way: $\Delta W_{EE} = P_i \cdot ctg(\alpha) = P_i \cdot \beta$.

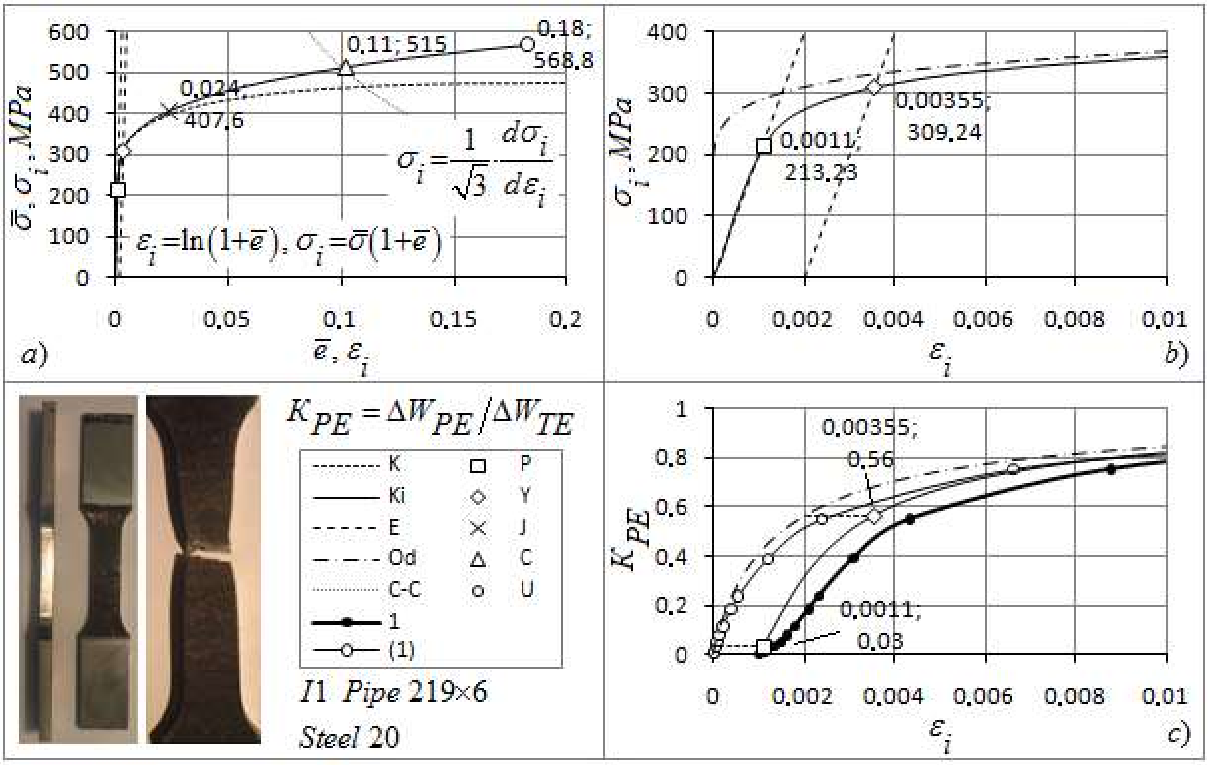

Figure 3. Diagram of deformation of steel 20 and relationship of deformation K_{PE} for thin-wall cylinder by the example of I1 sample:

K is the tension diagram of the sample cut out in circumferential direction; Ki is the actual deformation diagram plotted on K using dependencies (4); E is the elastic deformations; Od is the stresses as function form Odkvist's parameter; C-C is the dependence $d\sigma_i / (\sqrt{3} d\varepsilon_i)$; 1, (1) is the complete and residual deformations on experimental data in accordance with formula (13); J is the values till which I1 sample was tested in WJ; the other designations correspond Figure 1.

Figure 3 provides the tension diagrams (K) for the sample cut out in circumferential direction of pipe (initial state) from which sample I1 was made, and plotted based on it using dependencies (4) the actual deformation diagram (Ki). Diagram (K) and, respectively, (Ki) are presented before deformations corresponding to tensile strength, i.e. start of neck formation. Since for cylinder σ_t in times more than σ_z, then all the calculations were carried out on (Ki) diagram. It should be noted that the diagram in axial direction in contrast to circumferential has a yield segment and the calculations carried using it provide bad description of the test results. Analysis of formula (14) shows that the calculation pressure is linearly related with initial thickness of wall. Since actual thickness of wall has some spread characterized with thickness variation then a value being responsible for plastic deformations can be taken as s_o. This value, as a rule, is somewhat lower than average. In our calculations (see 4 Figure 2) nominal thickness, i.e. 6 mm, was taken as s_o.

Figures 2 and 3c show that if intensity of stresses in cylinder wall reaches σ_{02} level then the coefficient of permanent expansion amounts to sufficiently considerable values. Also is should be noted that a kink on the diagram of loading with internal pressure (see value P_Y on Figure 4b) takes place very close to this level. The calculation values corresponding to σ_{02} on Figures 4a and 4c virtually match with the value corresponding to P_Y.

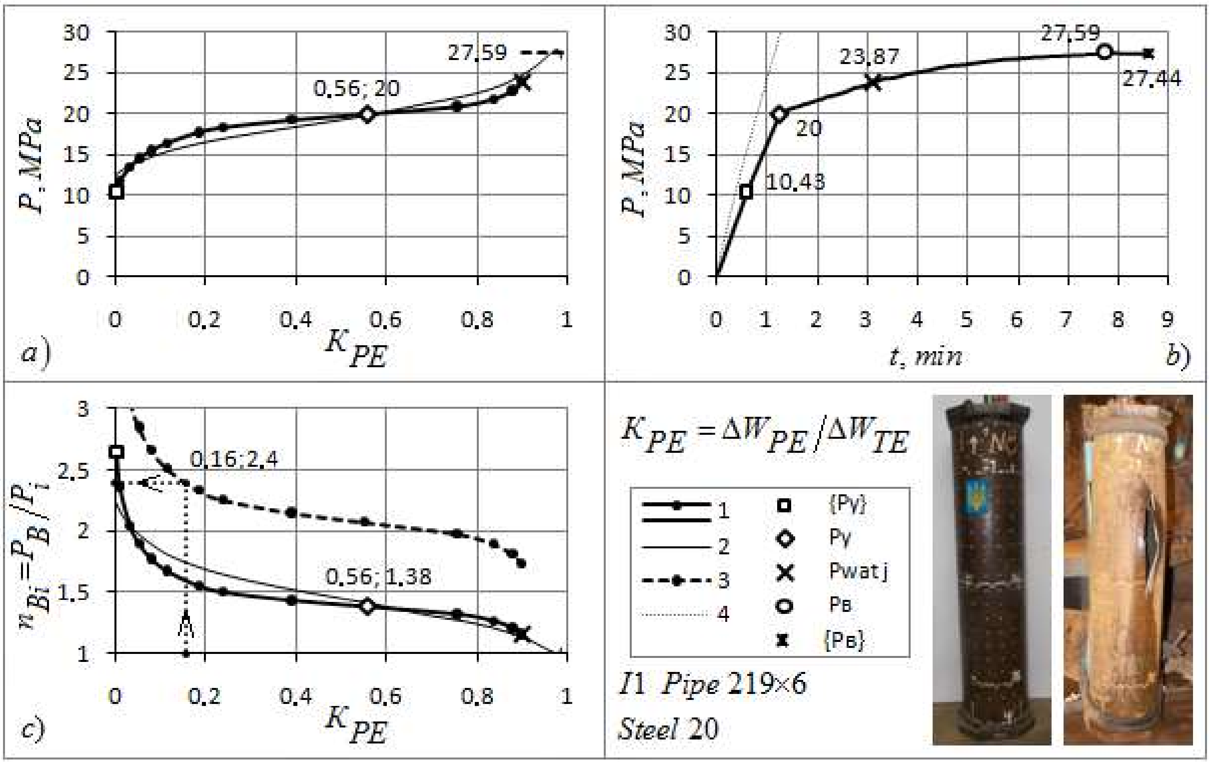

Figure 4. Dependence between pressure, coefficient of permanent expansion and current coefficient of strength margin, by the example of sample I1:

a) dependence between pressure, coefficient of permanent expansion; b) envelope of displacement diagrams of sample loading with internal pressure in time; c) dependence between the coefficient of permanent expansion and current coefficient of strength margin.

1 – experimental data; 2 – calculation data; 3 – ordinates of dependence 1 multiplied by 1.5; 4 – dependence for hydraulic receivers without object of testing.

In the area of large plastic deformations, deformations are proportional to the sample loading time, (the loading rate of the pump is close to constant). It is also interesting to note that the fracture pressure turned out to be less than the maximum pressure that the sample withstood, see figure 4b: $\{P_B\} < P_B$. This is in complete agreement with the provisions on loss of uniform plastic deformation. If the cylinder, for example, failures without reaching $dP = 0$ condition, then this indicates that it has been «work» of some defect or loading rate is too big. After failure of I1 sample, the residual deformations of the perimeter in its central part made 0.082. If they are used for intensity evaluation, then its equals to 0.095 that is very close to C on Figure 3a.

The experimental curve $n_{Bi}=f\left(K_{PE}\right)$ is the dependence between the level of plastic deformations and cylinder limiting state. It is an integral curve accumulating the properties of material and design peculiarities of the cylinder. The design peculiarities are shape of the bottom and neck, level of out-of-roundness and variation in thickness, bending of the axis, allowable cavities etc. Nevertheless, it can be seen that it is close to theoretical curve, see Figure 4c. Also it should be noted that theoretical curves plotted on diagrams I and II (see Figure 1b) match between themselves. However, they demonstrate sufficient difference at the initial stage from the experimental one, also matching between themselves for oxygen cylinders curves I and II. Such mismatch is apparently related with the fact that the tension samples were cut out from under shoes, and not to full extent characterize the properties of cylinder wall. The similar results have two other oxygen cylinders III and IV made of steel 35G.

In the case of periodic check of the cylinders made on type of sample I1 using dependence 1 (see Figure 4c) it is possible to determine the coefficient of static strength margin, but in relation to hydraulic test pressure. Since for sample I1 relationship $P_H / P_W = 1.5$ then the ordinates of the dependencies 3 were obtained from ordinates of experimental dependency 1 by multiplying for 1.5 Dependence 3 can serve as a diagram of evaluation of the static strength margin on the coefficient of permanent expansion in the case of periodic check of the cylinders of this type.

4. Conclusions

The deformation theory of plasticity can be used in calculation of thin-wall cylinder. Accepting the Poisson's ratio equal 0.5 the calculations are significantly simplified.
Using the actual diagram of steel deformation, taken from uniaxial tension diagram, it is possible to get precise relationship of stresses and deformations in wall of the cylinder, at its loading with internal pressure up to limiting state (failure).
Theoretical dependence between the coefficient of static strength margin and permanent expansion of one type cylinders does not depend on geometry parameters.

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References

1. GOST 949-73. Steel cylinders of small and average volume for gases for Pp ≤ 19.6 MPa (200 kg/cm2).
2. Reliability and economic feasibility of high-pressure cylinder produced in Ukraine/ E. F. Garf, R. I. Dmytriienko, A. A. Perepichay, A. P. Dengin, V. A. Pletnev // Techn. Diagn. and Non-Destructive Testing. – 2012 – No.2 – 36-41 pages. – (in Russian) UDK 620.19.14, URL: http://new-idea.kulichki.net/pubfiles/120918213704.pdf.
3. GOST 1497-84 (ISO 6892-84) – Metals. Methods of tension test.
4. ISO 9809-1:2010(E). Gas cylinders – Refillable seamless steel gas cylinders – Design, construction and testing – Part 1: Quenched and tempered steel cylinders with tensile strength less than 1 100 MPa.
5. ISO 7866:1999. Gas cylinders – Refillable seamless aluminium alloy gas cylinders – Design, construction and testing. (This standard has been revised by: ISO 7866:2012).
6. ISO 6406:2005(E). Gas cylinders. Seamless steel gas cylinders. Periodic inspection and testing.
7. BS EN 1802:2002. Transportable gas cylinders. Periodic inspection and testing of seamless aluminium alloy gas cylinders.
8. CGA C-5: Cylinder Service Life-Seamless Steel High Pressure Cylinders.
9. Dmytriienko R. I. Elastic volume change of seamless steel cylinders, «K-factor», stress limitation // Working note: RZ DRI – 030816. (In Russian).
   URL: http://www.dri1.cc.ua/RZ/RZ-DRI-030816.pdf.
10. Residual expansion of cylinders (brief review) / R. I. Dmytriienko, E. F. Garf, V. P. Chizhichenko / / Tech. diagnostics and non-destructive testing. – 2014. – No.1. – p. 23-28. – UDK 620.19.30. (In Russian).
    URL: http://new-idea.kulichki.net/pubfiles/140214224111.pdf.
11. «Working notes» on the website www.dri1.cc.ua.
12. Obraztsov I. F., Vasilyev V. V., Bunakov V. A. Optimal reinforcement of shells of revolution from composite materials. M., «Mashinostroenie», 1977, 144 p. (In Russian).
13. Malinin N. N. Applied theory of plasticity and creep. Textbook for university students. 2nd Ed., reviewed. M., «Mashinostroenie», 1975. – 400 p. with pictures. (In Russian).
14. «Hydraulic testing» playlist on the «ROMAN DMYTRIIENKO» channel of YouTube video hosting.